Supplementary Information for
"Antilocalization of Coulomb Blockade in a Ge-Si Nanowire"

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Supplementary material for "Antilocalization in a Coulomb Blockaded Ge-Si Nanowire", is given on the following topics:

1. Applicability of the theory
2. Using V_4 to perturb the quantum dot
3. Magnetoconductance fit details

1. Applicability of the theory

This section discusses the applicability of the Coulomb blockade peak distributions [Eqs. (2-4), in the main text] to the experiment.

These equations require $\Gamma < kT < \Delta$. $\Gamma, kT < \Delta$ ensures transport through single energy levels, and $\Gamma < kT$ ensures that the rate equations apply [1, 2]. The temperature of the quantum dot is $kT = 60$ μeV, determined from Coulomb blockade thermometry. Note that $T > T_{\text{base}}$ because of the large lock-in excitations used when gathering Coulomb peak statistics. The average tunnel rate is $\overline{\Gamma} = 1$ μeV, determined from the zero-field average Coulomb peak height at zero field using $\overline{g}_p = \chi_{s=2} \frac{2e^2}{\hbar} \frac{\overline{\Gamma}}{kT} \overline{\alpha}$. The mean level spacing is $\Delta = 0.2$ meV, determined from Coulomb blockade spectroscopy and confirmed by temperature dependence (Fig. 1, Fig. 2(c) in main text). The requirement $\Gamma < kT < \Delta$ is therefore squarely satisfied in this experiment.

Equations (2-4) also require the quantum dot to be diffusive or chaotic. The quantum dot studied here is diffusive. There are at least $N_H = 600$ holes in the dot, determined from counting Coulomb oscillations. The length of the quantum dot lies in the range $L = 200 - 600$ nm, corresponding to the length of the middle segment and the entire wire. This implies $M = 4w/\lambda_F = 4 - 6$ occupied transverse modes. Here we have used the three-dimensional expression to estimate the Fermi wavelength $\lambda_F = 2L(\pi w)^2(3N_H)^{-1/3} \sim 6 - 9$ nm (justified by $M \gg 1$). The Drude elastic scattering length is $l = h\mu/\lambda_F e = 35 - 50$ nm. The mobility $\mu = 800$ cm^2/Vs, determined from the slope of the pinch-off curve in Fig. 4 inset and the Drude relation $g = \pi w^2 \mu n e/(4L)$, is consistent with previous estimates under similar conditions [3, 4]. The dot therefore satisfies $l < L$ and is diffusive.

2. Using V_4 to perturb the quantum dot

This section describes the method of changing V_4 to increase the number of statistically independent Coulomb peaks. As shown in Fig. S1, the Coulomb peak heights evolve as a function of V_4, and eventually become uncorrelated with their original values. We examine the peaks at $V_4 = 0.2$ V and $V_4 = -0.1$ V. The number of peaks that can be acquired using this technique is eventually limited by the fact that large excursions in V_4 significantly change the tunnel rates to the leads.

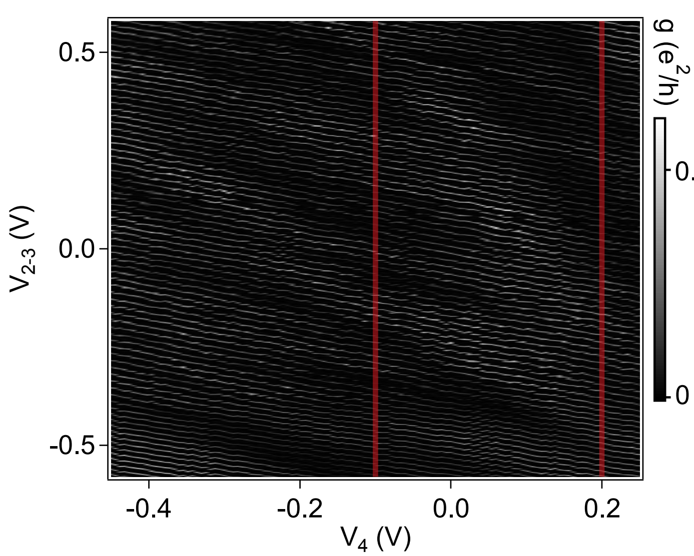

FIG. S1. Device conductance g as a function of gate voltages V_{2-3} and V_4. The red lines indicate the values of V_4 used for the Coulomb peak ensemble in the main text.

3. Magnetoconductance fit details

In this section we discuss the magnetoconductance fit (Fig. 4 main text) in more detail. Figure S2 shows the magnetoconductance data along with theory curves for different parameter values (summarized in Table I). The

purple curve, corresponding to $l_{so} = 10$ nm is the fit shown in the main text. The inset displays the minimal χ^2 value obtained at each l_{so}, fit with the constraint $l_e < 10$ μm. While the presence of correlated errors due to $1/f$ device noise excludes the use of a formal χ^2 analysis, we interpret the inset as indicating that the data are consistent with $l_{so} < 20$ nm.

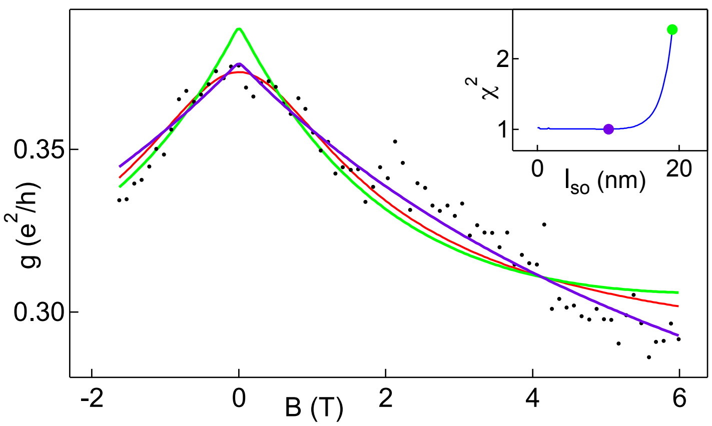

FIG. S2. Device conductance, g, as a function of magnetic field, B. The data are identical to Fig. 4 in the main text. The green and purple curves correspond to $l_{so} = 19$ nm and $l_{so} = 10$ nm with g_∞, l_e, and l_ϕ treated as fit parameters. The red curve is a fit assuming perfectly specular boundary scattering with l_{so}, g_∞, l_ϕ treated as fit parameters. Fit parameters for all curves are given in Table I. *Inset:* Goodness of fit, χ^2, as a function of l_{so} with g_∞, l_e, and l_ϕ treated as fit parameters. The dots correspond to the l_{so} values of the curves in the main portion of the figure.

description	color	l_{so} (nm)	g_∞ (e^2/h)	l_e (nm)	l_ϕ (nm)
$l_{so} = 10$ nm	purple	fixed	0.25	10^4	610
$l_{so} = 19$ nm	green	fixed	0.67	10^4	1250
specular	red	1.4	0.39	15	190

TABLE I. Parameters for the curves in Fig. S2.

[1] C. W. J. Beenakker, Physical Review B **44**, 1646 (1991).
[2] Y. Ahmadian and I. Aleiner, Physical Review B **73**, 073312 (2006).
[3] W. Lu, Proceedings of the National Academy of Sciences **102**, 10046 (2005).
[4] X.-J. Hao, T. Tu, G. Cao, C. Zhou, H.-O. Li, G.-C. Guo, W. Y. Fung, Z. Ji, G.-P. Guo, and W. Lu, Nano Letters **10**, 2956 (2010).