Supplementary Figure 1. Atomic force microscopy (AFM) images of Al and SiO$_2$/Al film. (a) Al film; (b) SiO$_2$/Al film. Left panel is the original AFM image. Right panel is height cross section along the yellow line in left panel. The bottom panel gives the statistical data of surface roughness.
Supplementary Figure 2. Photonic mode vs. Hybrid photonic/plasmonic modes. a. Panels from left to right are the electric field distribution $|E(x, y)|$ of HE11-like and EH11-like photonic modes in GaN-SiO$_2$ configuration for $a = 210$ nm, TE01-like and TM01-like modes for $a = 270$ nm, and EH11-like/TM01-like modes around cut-off size, respectively. b. The electric field distributions $|E(x, y)|$ of four typical hybrid modes in GaN-SiO$_2$-Al configuration for the GaN nanowire edge length of 180 nm. The calculated propagation lengths for the four modes are 0.85, 1.26, 0.84 and 1.49 μm, respectively, and the respective normalized mode areas $A_n/A_o$ with $A_o = (\lambda/2)^2$ are 0.10, 0.22, 0.22 and 0.25.

Supplementary Figure 3. Lasing in photonic laser device. The emission of a single GaN sitting on SiO$_2$/Si substrate as a function as pumping fluence is measured. The GaN nanowire has an edge length of 250 nm. The intensity when the lasing polarization is perpendicular to the nanowire axis $c$ is higher than the component with a polarization along $c$ (nanowire long axis).
Supplementary Figure 4. (a) The PL emission of GaN nanowire below lasing threshold. The center wavelength of emission peak when the polarization is along $z$ and $x$ is 368 and 372 nm, respectively. The emission peak position may be due to the two types of excitons with different polarization direction in GaN nanowires. (b-c) Effective gain distribution (in arbitrary unit) in the GaN nanowire pumped by laser at 355 nm polarized perpendicular (b) or parallel (c) to the nanowire axis.
Supplementary Discussion

The dipole-field coupling efficiency:

We consider a single linearly polarized dipole emitter located at \((x, y)\) in the GaN nanowire region. \(\mathbf{p}(x, y) = p(x, y)\mathbf{u}\), with \(\mathbf{u} = \mathbf{x}, \mathbf{y}, \text{ or } \mathbf{z}\) being a unit vector. The dipole emitter drives the hybrid plasmonic modes that propagate along +z and -z direction. The dipole-field coupling efficiency \(A_u(x,y)\) is defined as the ratio of the total energy of the launched hybrid plasmonic fields in both directions over the energy of the dipole in bulk semiconductor. It is calculated by the overlap integral between the electromagnetic field radiated by the dipole and that of the hybrid plasmonic mode.

Since there are only nontrivial \(y\) and \(z\) components for the hybrid plasmonic mode, only dipoles polarized along \(y\) or \(z\) direction could drive the hybrid plasmonic mode. Note that the distributions and intensities of dipole-field coupling efficiency for \(y\)-polarized and \(z\)-polarized dipoles are similar to those of the \(E_y\) and \(E_z\) components of the hybrid plasmonic mode, respectively. This is due to the fact that the larger electric field of the waveguide eigenmode at the position of the single dipole emitter, the larger energy transfer from the dipole to the waveguide field. In other words, the dipole-field coupling efficiency assembles the hybrid plasmonic mode because of sampling effect. These characteristics can be understood by the following equation\(^8\):

\[
\Gamma = \frac{2}{\hbar} \text{Im}[\mathbf{p} \cdot \mathbf{E(r)}] \\
\text{(S12)}
\]

where \(\Gamma\) is the spontaneous decay rate of the dipole, \(\mathbf{E}\) is the total electric field that satisfies Maxwell equations in the presence of the dipole, which can be expanded into the hybrid plasmonic mode and other radiation components. We should also note that the maximum dipole-field coupling efficiency is larger than 1.0 because of the plasmon enhancement.

The effective gain

It is widely accepted that the stimulated emission rate (and thus the gain) depends on the intensity of the excitation field, which is proportional to \(|\mathbf{E}|^2\). Here we further assume that the polarization of the dipoles is also parallel to the excitation electric field. As a result, for the pump laser polarized along \(z\) direction, the dipoles in the GaN region are expressed as
\[ p_z(x, y) \propto |E_z|^2 \]  \hspace{1cm} (S13)

where \( E_z \) is the nonzero \( z \) component of the total excitation field, as shown in Fig. 5d; for the pump laser polarized along \( x \) direction, the dipoles in the GaN region are expressed as

\[ p_x(x, y) \propto |E_x|^2, \quad p_y(x, y) \propto |E_y|^2 \]  \hspace{1cm} (S14)

where \( E_x \) and \( E_y \) are nonzero \( x \) and \( y \) components of the total excitation field, as shown in Fig. 5e and f, respectively.

The effective gain harvested by the hybrid plasmonic mode is then expressed as

\[ G_{\text{eff}} \propto A_z |E_z|^2 \]  \hspace{1cm} (S15)

for the pump laser polarized along \( z \) direction, and

\[ G_{\text{eff}} \propto A_x |E_x|^2 + A_y |E_y|^2 \]  \hspace{1cm} (S16)

for the pump laser polarized along \( x \) direction.

The total effective gain is then obtained by integrating over the whole GaN nanowire region. The ratio of total effective gain in (b) over that in (c) is about 1.87, indicating the pump laser polarized parallel to the nanowire axis will result in higher lasing intensity than that polarized perpendicular to the nanowire axis, as shown in Supplementary Figure S5. The calculation result agrees well with the measured result shown in Fig. 5b and c, where the ratio of peak emission intensity for \( E_{\parallel} \) over that for \( E_{\perp} \) is about 1.46.